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HS/XII/A. Sc. Com/M/26

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions.

General Instructions :

Read the following instructions very carefully and strictly follow them.

- (i) This Question Paper contains 38 questions. **All** questions are compulsory.
- (ii) This Question Paper is divided into five Sections—A, B, C, D and E.
- (iii) Section—A comprises of 20 questions (Q. Nos. **1** to **20**) of 1 mark each. Question Nos. **11** to **20** are multiple choice questions (MCQs).

Section—B comprises of 5 questions (Q. Nos. **21** to **25**) of 2 marks each.

Section—C comprises of 6 questions (Q. Nos. **26** to **31**) of 3 marks each.

(2)

Section—D comprises of 3 questions (Q. Nos. **32** to **34**) of 4 marks each.

Section—E comprises of 4 questions (Q. Nos. **35** to **38**) of 5 marks each.

(iv) There is no overall choice. However an internal choice has been provided in 3 questions in Section—B, 3 questions in Section—C, 1 question in Section—D and 3 questions in Section—E.

(v) Use of calculator is not allowed.

SECTION—A

(Marks : 20)

1. Find AB , if $A = \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$. 1

2. Find the value of k , if the function defined by

$$f(x) = \begin{cases} kx^2, & \text{if } x < 1 \\ 5, & \text{if } x \geq 1 \end{cases}$$

is continuous at $x = 1$. 1

3. Find $\frac{dy}{dx}$ for the following : 1

$$2x + 3y = \sin y$$

(3)

4. Find $\frac{dy}{dx}$, if $x = a \cos \theta$ and $y = a \sin \theta$. 1
5. Evaluate $\int_0^1 \frac{dx}{x^2}$. 1
6. Verify that $y = e^x - 1$ is a solution of the differential equation $y' - y = 0$. 1
7. Find the anti-derivative of $3x^2 - 4x^3$. 1
8. Evaluate $\int_1^2 \sin^5 x \cos^4 x \, dx$. 1
9. Prove that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 2x$, is both one-one and onto. 1
10. Find the unit vector in the direction of the vector $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$. 1

Choose the correct answer :

11. Let R be a relation in the set $\{1, 2, 3, 4\}$ given by $R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$. Then
- (a) R is reflexive and symmetric but not transitive
- (b) R is reflexive and transitive but not symmetric
- (c) R is symmetric and transitive but not reflexive
- (d) R is an equivalence relation 1

(4)

12. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = x^4$. Then

- (a) f is one-one and onto
- (b) f is many-one and onto
- (c) f is one-one but not onto
- (d) f is neither one-one nor onto

1

13. The principal value of $\cos^{-1} \frac{\sqrt{3}}{2}$ is

- (a) $\frac{\pi}{6}$
- (b) $\frac{\pi}{3}$
- (c) $\frac{\pi}{4}$
- (d) $\frac{\pi}{2}$

1

14. If $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$, then the value of x is

- (a) $2\sqrt{2}$
- (b) $\sqrt{2}$
- (c) 2
- (d) -2

1

(5)

15. The order and degree of the differential equation

$$\frac{dy}{dx} + 3y \frac{d^2y}{dx^2} = 0$$
 are respectively

(a) 2 and 4

(b) 4 and 2

(c) 2 and 1

(d) 1 and 2

1

16. The rate of change of area of a circle with respect to its radius r at $r = 6$ cm is

(a) 10 cm

(b) 12 cm

(c) 8 cm

(d) 11 cm

1

17. The magnitude of the vector $\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$ is

(a) 0

(b) 3

(c) 1

(d) 1

1

(6)

18. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) \cdot \hat{j} \cdot (\hat{i} \times \hat{k}) \cdot \hat{k} \cdot (\hat{i} \times \hat{j})$ is

(a) 0

(b) 1

(c) -1

(d) 3

1

19. If $P(A) = \frac{1}{2}$ and $P(B) = 0$, then $P(A|B)$ is

(a) 0

(b) $\frac{1}{2}$

(c) 1

(d) Not defined

1

20. Let E and F be events with $P(E) = \frac{1}{3}$, $P(F) = \frac{1}{2}$ and

$P(E \cap F) = \frac{1}{6}$. Then

(a) E and F are independent events

(b) E and F are mutually exclusive events

(c) E and F are disjoint events

(d) None of the above

1

(7)

SECTION—B

(Marks : 10)

21. Simplify :

2

$$\begin{array}{cccccc} \cos & \cos & \sin & \sin & \sin & \cos \\ & \sin & \cos & & \cos & \sin \end{array}$$

Or

Show that the matrix

$$A = \begin{pmatrix} 1 & 1 & 5 \\ 1 & 2 & 1 \\ 5 & 1 & 3 \end{pmatrix}$$

is a symmetric matrix.

22. Find the value of k if the area of a triangle whose vertices are $(k, 0)$, $(4, 0)$ and $(0, 2)$ is 4 square units.

2

Or

$$\text{If } A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 3 \\ 5 & 4 & 9 \end{pmatrix}, \text{ then find } |A|.$$

23. Is the function defined by

$$f(x) = \begin{cases} x - 5, & \text{if } x < 1 \\ x + 5, & \text{if } x \geq 1 \end{cases}$$

a continuous function at $x = 1$?

2

(8)

Or

Find the point of discontinuity of the function f defined by

$$f(x) = \begin{cases} 2x - 3, & \text{if } x \geq 2 \\ 2x + 3, & \text{if } x < 2 \end{cases}$$

24. Evaluate $\frac{(\log x)^2}{x} dx$. 2

25. If $P(A) = 0.8$, $P(B) = 0.5$ and $P(B|A) = 0.4$, then find—

(a) $P(A \cap B)$

(b) $P(A \cup B)$ 2

SECTION—C

(Marks : 18)

26. Evaluate $\frac{xe^x}{(1-x)^2} dx$. 3

Or

Evaluate $\frac{2x}{x^2 - 3x - 2} dx$.

27. The radius of a circle is increasing uniformly at the rate of 3 cm/s. Find the rate at which the area of the circle is increasing when the radius is 10 cm. 3

(9)

- 28.** Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. 3

Or

Find the angle between the vectors $\hat{i} + 2\hat{j} + 3\hat{k}$ and $3\hat{i} + 2\hat{j} + \hat{k}$.

- 29.** Find the equation of the line in vector and Cartesian form that passes through the point with position vector $2\hat{i} + \hat{j} + 4\hat{k}$ and in the direction of $\hat{i} + 2\hat{j} + \hat{k}$. 3

- 30.** A particle moves along the curve $6y = x^3 + 2$. Find the points on the curve at which the y -coordinate is changing 8 times as fast as the x -coordinate. 3

- 31.** Evaluate $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} \sqrt{\sin x}} dx$ by using the properties of definite integrals. 3

Or

Prove that $\int_1^x x^{17} \cos^4 x \, dx = 0$.

SECTION—D

(Marks : 12)

- 32.** Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$. 4

(10)

Or

Find the area enclosed by the circle $x^2 + y^2 = a^2$.

33. Solve graphically : 4

Maximize $Z = 5x + 3y$
subject to

$$\begin{aligned} 3x + 5y &= 15 \\ 5x + 2y &= 10 \\ x &\geq 0, y \geq 0 \end{aligned}$$

34. In each of the following cases, state whether the function is one-one, onto or bijective. Justify your answer : 4

(a) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 3 + 4x$

(b) $f : \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = 1 + x^2$

SECTION—E

(Marks : 20)

35. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. 5

Or

Show that the semi-vertical angle of the right circular cone of the maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.

(11)

- 36.** Find the shortest distance between the lines

$$\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$$

and $\vec{r} = (2\hat{i} + \hat{j} + \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

5

Or

Find the vector equation of the line passing through the point (1, 2, 4) and perpendicular to the two lines $\frac{x-8}{3} = \frac{y-19}{16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{5}$.

- 37.** Solve the following system of linear equations using matrix method :
- 5

$$\begin{array}{rcl} x + y + z & = & 4 \\ 2x + y + 3z & = & 0 \\ x + y + z & = & 2 \end{array}$$

Or

Verify $A(\text{adj}A) = (\text{adj}A)A = |A|I$ for the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix}$$

- 38.** There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the time and third is an unbiased coin. One of the three coins is chosen at random and tossed, it shows heads. What is the probability that it is the two-headed coin?
- 5

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