

2023**MATHEMATICS***Full Marks : 80**Time : 3 hours**General Instructions :*

1. All questions are compulsory.
2. The question paper consists of 36 questions divided into four Sections A, B, C and D.
Section A consists of 20 questions of 1 mark each.
Section B consists of 6 questions of 2 marks each.
Section C consists of 6 questions of 4 marks each.
Section D consists of 4 questions of 6 marks each.
3. There is no overall choice. However there will be internal choices for 4 marks questions and 6 marks questions.
4. Use of calculator is not permitted.

SECTION – A

1. If R is the set of real numbers and Q is the set of rational numbers, then what is $R - Q$? 1
2. Write the power set of $A = \{1, 2, 3\}$. 1

3. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y . 1

4. Find the degree measure corresponding to the radian measure : $\frac{11}{16} \cdot \left[\text{use } \pi = \frac{22}{7} \right]$. 1

5. Find the value of $\cos(-1410^\circ)$. 1

6. Express the complex number i^{19} in the form $(a + ib)$. 1

7. Solve the equation $x^2 + 3x + 9 = 0$. 1

8. Solve : $24x < 100$, when x is an integer. 1

9. Write the first five terms of the sequence whose n^{th} term is $a_n = \frac{2n - 3}{6}$. 1

10. If $\frac{2}{7}, x, \frac{7}{2}$ are in Geometric Progression, find the value of x . 1

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11. Find the equation of the line through $(-2, 3)$ with slope -4 . 1
12. Find the equation of the line through the points $(3, -2)$ and $(-1, 4)$. 1
13. Find the intercepts cutoff by the straight line $3x + 2y = 6$ from the co-ordinate axes. 1
14. Find the equation of the circle with centre $\left(\frac{1}{2}, \frac{1}{4}\right)$ and radius $\frac{1}{12}$. 1
15. Evaluate : $\lim_{x \rightarrow 0} \left(\frac{\sin ax}{\sin bx} \right)$; $a, b \neq 0$. 1
16. Evaluate : $\lim_{x \rightarrow 0} \left\{ \frac{(x+1)^5 - 1}{x} \right\}$. 1

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17. Write down the sample space when a coin is tossed three times. 1
18. One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be a diamond. 1
19. Find the number of words that could be formed with the letters of the word "COLLEGE". 1
20. If $\frac{1}{6!} + \frac{1}{7!} = \frac{x}{8!}$, find x . 1

SECTION – B

21. Find the value of $\tan 15^\circ$. 2
22. Find the multiplicative inverse of $(\sqrt{5} + 3i)$. 2
23. Find r , if ${}^5P_r = {}^6P_{r-1}$. 2
24. Find the Co-efficient of x^5 in $(x+3)^8$. 2

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25. Using binomial theorem, evaluate $(99)^5$. 2

26. Find the derivative of $\frac{1}{x^2}$. 2

SECTION – C

27. The relation f is defined by

$$f(x) = \begin{cases} x^2, & 0 \leq x \leq 3 \\ 3x, & 3 \leq x \leq 10 \end{cases}$$

The relation g is defined by

$$g(x) = \begin{cases} x^2, & 0 \leq x \leq 2 \\ 3x, & 2 \leq x \leq 10 \end{cases}$$

Show that f is a function and g is not a function. 4

28. Solve the following system of inequalities graphically:
 $x + 2y \leq 10$, $x + y \geq 1$, $x - y \leq 0$, $x \geq 0$, $y \geq 0$. 4

29. Find the co-ordinates of the foci, the vertices, the eccentricity and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$. 4

Or

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Find the equation of the ellipse whose major axis on the x -axis and passes through the points $(4, 3)$ and $(6, 2)$.

30. Find the co-ordinates of the points which trisect the line segment joining the points $P(4, 2, -6)$ and $Q(10, -16, 6)$. 4

Or

If the origin is the centroid of the triangle PQR with vertices $P(2a, 2, 6)$, $Q(-4, 3b, -10)$ and $R(8, 14, 2c)$, then find the values of a , b and c .

31. Find the derivative from first principle of $\sin x$ 4

Or

Find the derivatives of

(i) $\frac{2}{(x+1)}$ (ii) $\log \sin x$

32. In class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology. 4

Or

Three coins are tossed once. Find the probability of getting (i) 3 heads (ii) at least 2 heads.

SECTION – D

- 33.** A college awarded 38 medals in football, 15 in basketball and 20 in cricket. If these medals went to a total of 58 men and only three men got medals in all the three sports, how many received medals in exactly two of the three sports.

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- 34.** The ratio of the A.M. and G.M. of two positive numbers a and b is $m : n$.

Show that $a : b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$.

6

Or

Insert five numbers between 8 and 26 such that the resulting sequence is an A.P.

- 35.** Prove that

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(i) $\cot 4x (\sin 5x + \sin 3x) = \cot x (\sin 5x - \sin 3x)$

(ii) $\tan 4x = \frac{4 \tan x (1 - \tan^2 x)}{1 - 6 \tan^2 x + \tan^4 x}$

- 36.** Find the mean deviation about the mean for the following data:

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Marks obtained	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Number of Students	2	3	8	14	8	3	2

Or

Find the mean deviation about the median for the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60
Number of girls	6	8	14	16	4	2

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