

**HS/XI/A. Sc. Com/M/22****2022****MATHEMATICS***Full Marks : 80**Time : 3 hours**General Instructions :*

1. All questions are compulsory.
2. The question paper consists of 36 questions divided into four Sections A, B, C and D.  
Section A consists of 20 questions of 1 mark each.  
Section B consists of 6 questions of 2 marks each.  
Section C consists of 6 questions of 4 marks each.  
Section D consists of 4 questions of 6 marks each.
3. There is no overall choice. However there will be internal choices for 4 marks questions and 6 marks questions.
4. Use of calculator is not permitted.

**SECTION – A**

1. Write the set in the set-builder form  $\{1, 4, 9, \dots, 100\}$ .
2. Write the set  $\{x : x \in \mathbb{R}, -4 < x \leq 6\}$  as interval.

3. If  $A = \{3, 6, 9, 12, 15, 18, 21\}$   
 $B = \{4, 8, 12, 16, 20\}$   
find (i)  $A \cap B$  and (ii)  $A - B$ .

4. Find the modulus of the complex number  $-2 + \sqrt{5}i$

5. Solve  $3x + 8 > 2$  when  $x$  is an integer.

6. If  ${}^nC_8 = {}^nC_2$ , Find  ${}^nC_2$ .

7. Find the slope of the straight line parallel to the straight line passing through the points  $(-2, 3)$  and  $(8, -5)$ .

8. Find the equation of a circle with centre  $(-2, 3)$  and radius 4.

9. If the set of natural numbers is the universal set, then find the complement of the set  
 $\{x : x \text{ is a prime number}\}$

10. Find the radian measure of  $240^\circ$

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11. Write the contrapositive and converse of the statement.  
“If  $x$  is a prime number, then  $x$  is odd.”
12. Write the following statement in the form “if-then”  
“A quadrilateral is a parallelogram if its diagonals bisect each other.”
13. Find the median of the data 6, 7, 10, 12, 13, 4, 8, 12.
14. Write the general term in the expansion of  $(x^2 - y)^6$ .
15. If the set A has 3 elements and the set  $B = \{ 3, 4, 5 \}$ , then find the number of elements in  $A \times B$ .
16. Find the domain and range of the relation R defined by  
 $R = \{(x, x + 5) : x \in \{ 0, 1, 2, 3, 4, 5 \}\}$
17. Find the  $n^{\text{th}}$  term of the series  
 $3 \times 1^2 + 5 \times 2^2 + 7 \times 3^2 + \dots$

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18. Evaluate  $\lim_{x \rightarrow 2} \frac{x^3 - 8}{x - 2}$
19. How many 3 digit numbers can be formed from the digits 1, 2, 3, 4 and 5 assuming the repetition of the digits is allowed?
20. If E and F are events such that  $P(E) = \frac{1}{4}$ ,  $P(F) = \frac{1}{2}$  and  
 $P(E \cap F) = \frac{1}{8}$  find  $P(E \text{ or } F)$ .

#### SECTION – B

21. Prove that  
$$\cos\left(\frac{\pi}{4} - x\right)\cos\left(\frac{\pi}{4} - y\right) - \sin\left(\frac{\pi}{4} - x\right)\sin\left(\frac{\pi}{4} - y\right) = \sin(x + y)$$
22. Express the complex number  
 $\left(\frac{1}{3} + 3i\right)^3$  in the form  $a + ib$ .
23. Show that the points P(-2, 3, 5), Q(1, 2, 3) and R (7, 0, -1) are collinear.

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- 24.** Find the sum of odd integers from 1 to 2001.
- 25.** Find the co-ordinates of the focus, the vertex, the axis and the length of the latus rectum of the parabola  $y^2 = 12x$ .
- 26.** One card is drawn from a well shuffled deck of 52 cards. If each outcome is equally likely, calculate the probability that the card will be
- (i) not an ace
  - (ii) not a black card.

SECTION – C

- 27.** Prove that

$$2 \cos \frac{\pi}{13} \cos \frac{9\pi}{13} + \cos \frac{3\pi}{13} + \cos \frac{5\pi}{13} = 0$$

Or

Prove that

$$\frac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x} = \cot 3x$$

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- 28.** The sum of first three terms of a G.P. is 16 and the sum of next three terms is 128. Determine the first term and the sum of  $n$  terms of the G.P.

Or

Find the sum to  $n$  terms of the series whose  $n^{\text{th}}$  term is given by  $n^2 + 2^n$ .

- 29.** Convert the complex number  $-1-i$  into polar form.

Or

In how many ways one can select a cricket team of eleven players from 17 players in which only 5 bowlers can bowl if each cricket team of 11 must include exactly 4 bowlers.

- 30.** Find the equation of the hyperbola whose foci are  $(\pm 5, 0)$  and the transverse axis is of length 8 units.

Or

Find the equation of the ellipse with foci  $(\pm 5, 0)$  and the length of major axis is 26 units.

- 31.** Find the general solution of the equation  $\cos 4x = \cos 2x$ .

- 32.** Find the derivative of  $\frac{\sin x + \cos x}{\sin x - \cos x}$ .

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SECTION – D

- 33.** Using the principle of mathematical induction prove

that  $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} = 1 - \frac{1}{2^n}$ .

- 34.** Solve the system of inequalities graphically

$$3x + 4y \leq 60, \quad x + 3y \leq 30, \quad x \geq 0, \quad y \geq 0.$$

*Or*

Find  $(x+1)^6 + (x-1)^6$ . Hence or otherwise evaluate  $(\sqrt{2}+1)^6 + (\sqrt{2}-1)^6$

- 35.** In a survey it was found that 21 people liked product A, 26 liked product B and 29 liked product C. If 14 people liked products A and B, 12 people liked products C and A, 14 people liked products B and C and 8 liked all the three products. Find how many liked products C only.

- 36.** Calculate the mean, variance and standard deviation for the following distribution

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

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*Or*

The diameters of circles (in mm) drawn in a design are given below

Diameters	33-36	37-40	41-44	45-48	49-52
No of Circles	15	17	21	22	25

Calculate the standard deviation and the mean diameter of the circle.

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