Total No. of Printed Pages —6

HS/XI/A. Sc. Com/M/19

2019

MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into four Sections A, B, C and D.

   Section A consists of 4 questions of *1* mark each.

   Section B consists of 8 questions of *2* marks each.

   Section C consists of 11 questions of *4* marks each.

   Section D consists of 6 questions of *6* marks each.
3. There is no overall choice. However internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 marks each.
4. Use of calculator is not permitted.

SECTION — A

1. Write the solution set of the equation $x^2 + x - 2 = 0$ in roster form.

2. Find the domain of the function $f(x) = \sqrt{9 - x^2}$

3. Solve: $24x < 100$ when x is a natural number.

4. Evaluate $\lim_{x \to 0} \frac{\sin 4x}{\sin 2x}$

SECTION – B

5. Find the multiplicative inverse of $2 - 3i$.

6. Find the co-efficient of x^5 in the expansion of $(x+3)^8$.

7. Solve: $2\cos^2 x + 3\sin x = 0$

8. Given below are two statements

p : 25 is a multiple of 5

q : 25 is a multiple of 8.

Write the compound statements connecting these 2 statements with 'AND' and 'OR'. In both cases, check the validity of the compound statement.

9. Find r if $5\ {}^4P_r = 6\ {}^5P_{r-1}$

10. Find the derivative of $\frac{2}{x+1} - \frac{x^2}{3x-1}$

11. A die is rolled. If E be the event 'die shows 4' and F be the event 'die shows even number', then are E and F mutually exclusive?

12. If A = set of rational numbers

B = $\{x \mid x^2 - 4x + 2 = 0\}$

Find A – B and B – A.

SECTION — C

13. State whether the following sets are equal or not

I= $\{x : x^2 - 5x + 7 = 0,\ x \in R\}$ and J = ϕ

14. Find the value of tan $\frac{\pi}{8}$.

15. PQR is a triangle with vertices P($2a$, 2, 6), Q(–4, $3b$, –10) and R(8, 14, $2c$). If the centroid lies at the origin, calculate the values of *a, b, c.*

Or

Show that the points (–2, 3, 5), (1, 2, 3) and (7, 0, –1) are collinear.

16. How many numbers can be formed with digits 1, 2, 3, 4, 3, 2, 1 so that odd digits always occupy the odd places?

Or

Solve the following system of inequalities graphically :

$3x + 4y \le 60,\ x + 3y \le 30,\ x \ge 0,\ y \ge 0$

17. By principle of mathematical induction prove that

$$1^3 + 2^3 + 3^3 + \ldots\ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

Or

For every positive integer n, prove that $7^n - 3^n$ is divisible by 4.

18. Find real θ, such that $\frac{3 + 2i\sin\theta}{1 - 2i\sin\theta}$ is purely real.

19. Find the derivative of cos$x$ from 1st principle.

20. Find the equation of circle passing through (0,0) and making intercepts *'a'* and *'b'* on the co-ordinate axes.

Or

Find the ratio in which the line segment joining the points (4, 8, 10) and (6, 10, –8) is divided by the yz - plane.

21. Find the sum of the sequence 7, 77, 777, 7777, ...to n terms.

22. In a class of 35 students, 24 like to play cricket and 16 like to play football. Also each student likes to play at least one of the two games. How many students like to play both games?

23. In class XI of a School, 40% students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

SECTION – D

24. Prove that

$$\cos 2x \cos\frac{x}{2} - \cos 3x \cos\frac{9x}{2} = \sin 5x \sin\frac{5x}{2}.$$

25. Find the equation of the circle passing through the points (4, 1) and (6,5) and whose centre is on the line $4x + y = 16$.

Or

Find the co-ordinates of the foci and the vertices, the eccentricity and length of latus rectum of the hyperbola

$$\frac{y^2}{9} - \frac{x^2}{27} = 1$$

26. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.

27. Determine whether the expansion of $\left(x^2 - \frac{2}{x}\right)^{18}$ will contain a term containing x^{10}.

28. Evaluate $\lim\limits_{x \to 0} \frac{e^x + e^{-x} - 2}{x^2}$

29. The variance of 20 observations is 5. If each observation is multiplied by 2, find the new variance of the resulting observations.

Or

Find the mean deviation about the mean for the following data:

Marks Obtained	10–20	20–30	30–40	40–50	50–60	60–70	70–80
No. of Students	2	3	8	14	8	3	2

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