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HS/XII/A.Sc.Com/M/OC/20

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MATHEMATICS

(Old Course)

Full Marks : 100

Time : 3 hours

General Instructions :

- (i) Write all the answers in the Answer Script.
- (ii) The question paper consists of three Sections—A, B and C.
- (iii) Section—A consists of 15 questions, carrying 2 marks each.
- (iv) Section—B consists of 10 questions, carrying 4 marks each, out of which 3 questions have internal choices.
- (v) Section—C has 5 questions, carrying 6 marks each, out of which 2 questions have internal choices.

SECTION—A

- 1.** Let S be the set of all real numbers and let R be a relation in S defined by

$$R \equiv \{(a, b) : (1 + ab) > 0\}$$

Show that R is reflexive and symmetric.

2

(2)

- 2.** Let

$$R = \left\{ \left(a, \frac{1}{a} \right) : a \in N \text{ and } 1 \leq a \leq 5 \right\}$$

List the elements of the above relation. Find the domain and range.

2

- 3.** Show that the function $f : \mathbb{R} \rightarrow \mathbb{R} : f(x) = |x|$ is neither one-one nor onto.

2

- 4.** Show that the operation $*$ on \mathbb{Z} , defined by

$$a * b = a + b + 1 \quad \forall a, b \in \mathbb{Z}$$

satisfies (a) closure property and (b) commutative law.

2

- 5.** Construct a 3×2 matrix whose elements are given by $a_{ij} = i + 2j$.

2

- 6.** Let

$$A = \begin{bmatrix} 2 & 3 & -1 \\ 0 & -5 & 7 \end{bmatrix}$$

Verify that $(A')' = A$.

2

- 7.** Show that $f(x) = x^3$ is continuous at the point $x = 2$.

2

- 8.** Differentiate $y = \tan \sqrt{x}$ with respect to x .

2

(3)

9. If $x^3 + y^3 = 3axy$, find $\frac{dy}{dx}$. 2

10. Find $\frac{dy}{dx}$, when $x = at^2$, $y = 2at$. 2

11. A stone is dropped into a quiet lake and the waves move in circles. If the radius of a circular wave increases at the rate of 4 cm/s, find the rate of increase in its area at the instant when its radius is 10 cm. 2

12. Evaluate : 2

$$\int x^2 e^x dx$$

13. Let

$$\vec{a} = 3\hat{i} + 2\hat{j} \quad \text{and} \quad \vec{b} = 2\hat{i} + 3\hat{j}$$

Is $|\vec{a}| = |\vec{b}|$? Is $\vec{a} = \vec{b}$? 2

14. If

$$\vec{a} = 3\hat{i} + \hat{j} - 4\hat{k} \quad \text{and} \quad \vec{b} = 6\hat{i} + 5\hat{j} - 2\hat{k}$$

find $\vec{a} \times \vec{b}$ and $|\vec{a} \times \vec{b}|$. 2

15. If E_1 and E_2 are two independent events such that $P(E_1) = 0.35$ and $P(E_1 \cup E_2) = 0.60$, find $P(E_2)$. 2

SECTION—B

16. Express the matrix

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

as the sum of a symmetric and a skew-symmetric matrix.

4

Or

If

$$A = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$

verify that $A^2 - 4A - I = 0$ and hence find A^{-1} .

4

17. Show that the semivertical angle of a right circular cone of given surface area and maximum volume is

$$\sin^{-1}\left(\frac{1}{3}\right)$$

4

18. Verify Rolle's theorem for the function $f(x) = e^x \cos x$ in

$$\left[\frac{-\pi}{2}, \frac{\pi}{2} \right]$$

4

Or

Using Lagrange's mean value theorem, find a point on the curve $y = \sqrt{x-2}$, defined in the interval $[2, 3]$, where the tangent is parallel to the chord joining the end points of the curve.

4

(5)

19. (a) The side of a square is increasing at the rate of 0.2 cm/s. Find the rate of increase of the perimeter of the square. 2

(b) Find the approximate value of the cube root of 127. 2

20. Water is leaking from a conical funnel at the rate of 5 cm³/s. If the radius of the base of the funnel is 5 cm and its altitude is 10 cm, find the rate at which the water level is dropping when it is 2.5 cm from the top. 4

21. (a) If $y = e^x(\sin x + \cos x)$, prove that

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = 0 \quad 2$$

(b) Solve the differential equation

$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0 \quad 2$$

22. (a) Evaluate : 2

$$\int_0^a \frac{dx}{\sqrt{ax - x^2}}$$

(b) Evaluate : 2

$$\int_0^{\sqrt{2}} \sqrt{2 - x^2} dx$$

(6)

Or

Evaluate :

4

$$\int_0^{\pi} \frac{x}{(a^2 \cos^2 x + b^2 \sin^2 x)} dx$$

23. Find the intervals in which the function

$$f(x) = -2x^3 - 9x^2 - 12x + 1$$

is (a) strictly increasing and (b) strictly decreasing.

4

24. A line passes through the point (3, 4, 5) and is parallel to the vector $(2\hat{i} + 2\hat{j} - 3\hat{k})$. Find the equations of the line in the vector as well as Cartesian forms.

4

25. Find the equations of the normals to the curve $3x^2 - y^2 = 8$ parallel to the line $x + 3y = 4$.

4

SECTION—C

26. Using matrices, solve the following system of equations : 6

$$\begin{aligned} \frac{2}{x} + \frac{3}{y} + \frac{10}{z} &= 4 \\ \frac{4}{x} - \frac{6}{y} + \frac{5}{z} &= 1 \\ \frac{6}{x} + \frac{9}{y} - \frac{20}{z} &= 2 \end{aligned}$$

(7)

Or

(a) Without expanding the determinants, prove that

$$\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix} \quad 3$$

(b) Solve for x : 3

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

27. Evaluate

$$\int_a^b \sin x \, dx$$

from the first principle. 6

Or

Find the area of the smaller region bounded by the

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the straight line $\frac{x}{a} + \frac{y}{b} = 1$. 6

28. (a) Find the angle between the lines whose direction ratios are 2, 3, 6 and 1, 2, 2. 2

(b) Show that the lines

$$\frac{x-5}{4} = \frac{y-7}{4} = \frac{z+3}{-5} \quad \text{and} \quad \frac{x-8}{7} = \frac{y-4}{1} = \frac{z-5}{3}$$

intersect each other. Also, find the point of their intersection. 4

- 29.** A factory has three machines X , Y and Z , producing 1000 bolts, 2000 bolts and 3000 bolts per day respectively. The machine X produces 1% defective bolts, Y produces 1.5% defective bolts and Z produces 2% defective bolts. At the end of the day, a bolt is drawn at random and it is found to be defective. What is the probability that this defective bolt has been produced by machine X ? 6
- 30.** A firm manufactures two types of products A and B and sells them at a profit of ₹ 5 per unit of type A and ₹ 3 per unit of type B . Each product is processed on two machines M_1 and M_2 . One unit of type A requires one minute of processing time on M_1 and two minutes of processing time on M_2 , whereas one unit of type B requires one minute of processing time on M_1 and one minute of processing time on M_2 . Machines M_1 and M_2 are respectively available for at most 5 hours and 6 hours in a day. Find out how many units of each type of products should the firm produce a day in order to maximize the profit. Solve the problem graphically. 6

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