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HS/XII/A. Sc. Com/M/23

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MATHEMATICS

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

- (i) All questions are compulsory.
- (ii) This question paper contains 36 questions divided into four Sections—A, B, C and D. Section—A comprises of 20 questions of 1 mark each, Section—B comprises of 6 questions of 2 marks each, Section—C comprises of 6 questions of 4 marks each and Section—D comprises of 4 questions of 6 marks each.
- (iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 3 questions of Section—D. You have to attempt only one of the alternatives in all such questions.
- (iv) Use of calculator is not permitted.

SECTION—A

- 1.** Define an equivalence relation. 1

Or

A relation R in the set N of natural numbers is defined as
 $R = \{(x, y) : y = x + 5 \text{ and } x \leq 4\}$. Find the range of R . 1

(2)

2. Find the principal value of $\cos^{-1} \frac{1}{2}$. 1

Or

Evaluate : 1

$$\cos^{-1} \frac{1}{2} - 2 \sin^{-1} \frac{1}{2}$$

3. Find the value of x , if

$$\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix} \quad 1$$

Or

If $A = \begin{pmatrix} 0 & 1 \\ 0 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 5 \\ 0 & 0 \end{pmatrix}$, find AB and BA . 1

4. What is an objective function of a linear programming problem? 1

5. Evaluate : 1

$$\int_0^{\pi/2} \cos 2x \, dx$$

Or

Evaluate : 1

$$\int_0^1 \frac{dx}{1-x^2}$$

(3)

6. Differentiate : 1

$$2\sqrt{\cot x}$$

Or

- Find $\frac{dy}{dx}$, if $y = \sec(\tan x)$. 1

7. Write the order and degree of the differential equation

$$\frac{ds}{dt}^4 - 3s \frac{d^2s}{dt^2} = 0 \quad 1$$

8. Prove that $y = Ax$ is a solution of the differential equation

$$xy = y, \quad (x \neq 0)$$

and A is a constant. 1

9. Show that the function $f : R \rightarrow R$ given by $f(x) = x^3$ is injective, where R is the set of real numbers. 1

Or

Show that the modulus function $f : R \rightarrow R$ given by $f(x) = |x|$ is not one-one, where R is the set of real numbers. 1

10. Find $\frac{dy}{dx}$, if $y = \log(\cos e^x)$. 1

Or

If $2x + 3y = \sin x$, find $\frac{dy}{dx}$. 1

(4)

11. If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cap B) = \frac{7}{11}$, find $P(A \cup B)$. 1

12. Let E and F be events with $P(E) = \frac{3}{5}$, $P(F) = \frac{3}{10}$ and $P(E \cap F) = \frac{1}{5}$. Are E and F independent? 1

13. Evaluate : 1

$$\int_{x^2=1} \frac{1}{x^2} dx$$

Or

Evaluate : 1

$$\int \tan^2 x dx$$

14. Compute the magnitude of the following vector : 1

$$\vec{a} = \frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}$$

15. Evaluate : 1

$$\int_{-\pi/2}^{\pi/2} \sin^7 x dx$$

16. Prove that the function

$$f(x) = x^3 - 3x^2 - 3x + 100$$

is increasing in R , where R is the set of real numbers. 1

(5)

Choose the correct answer :

17. If $\sin^{-1} x = y$, then

(a) $0 \leq y$

(b) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

(c) $0 \leq y$

(d) $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

1

18. If $A = \frac{\cos \theta}{\sin \theta}$, then $A = A \cdot I$, if the value of θ is

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c)

(d) $\frac{3\pi}{2}$

1

(6)

19. The anti-derivative of $\sqrt{x} - \frac{1}{\sqrt{x}}$ is equal to

(a) $\frac{1}{3}x^{\frac{1}{3}} - 2x^{\frac{1}{2}} + c$

(b) $\frac{2}{3}x^{\frac{2}{3}} - \frac{1}{2}x^2 + c$

(c) $\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + c$

(d) $\frac{3}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}} + c$

1

Or

The rate of change of the area of a circle with respect to its radius r at $r = 6$ cm is

(a) 10

(b) 12

(c) 8

(d) 11

1

20. The value of $\hat{i} \cdot (\hat{j} \times \hat{k}) - \hat{j} \cdot (\hat{i} \times \hat{k}) + \hat{k} \cdot (\hat{i} \times \hat{j})$ is

(a) 0

(b) 1

(c) 1

(d) 3

1

(7)

SECTION—B

21. Find x and y , if

$$\begin{vmatrix} 1 & 3 \\ 0 & x \end{vmatrix} = \begin{vmatrix} y & 0 \\ 1 & 2 \end{vmatrix} = \begin{vmatrix} 5 & 6 \\ 1 & 8 \end{vmatrix}$$

2

Or

If $A = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix}$, show that $|2A| = 4|A|$. 2

22. Find the value of k , if the function defined by

$$f(x) = \begin{cases} kx^2, & \text{if } x < 2 \\ 3, & \text{if } x \geq 2 \end{cases}$$

is continuous at $x = 2$. 2

Or

Show that $f(x) = 5x - 3$ is continuous at $x = 5$. 2

23. Evaluate : 2

$$\int x e^x dx$$

Or

Evaluate : 2

$$\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$$

(8)

24. If $y = 500e^{7x} - 600e^{-7x}$, show that

$$\frac{d^2y}{dx^2} = 49y \quad 2$$

25. Find $|\vec{a} - \vec{b}|$, if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 2\hat{k}$. 2

Or

Show that the points $A(2, 3, 4)$, $B(-1, -2, 1)$ and $C(5, 8, 7)$ are collinear. 2

26. Find $\frac{dy}{dx}$, if

$$y = \cos^{-1} \frac{1-x^2}{1+x^2}, \quad 0 < x < 1 \quad 2$$

Or

Evaluate : 2

$$\int \frac{x^2}{1-x^6} dx$$

(9)

SECTION—C

- 27.** Show that the function $f: R \rightarrow R$ defined by $f(x) = 3 - 4x$ is one-one and onto, where R is the set of real numbers. 4

Or

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$ is an equivalence relation. 4

- 28.** Find the intervals in which the function f given by

$$f(x) = 4x^3 - 6x^2 - 72x + 30$$

is (a) strictly increasing and (b) strictly decreasing. 4

Or

Sand is pouring from a pipe at the rate of $12 \text{ cm}^3/\text{s}$. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm? 4

- 29.** Using the properties of definite integrals, evaluate

$$\int_0^{\pi/4} \log(1 + \tan x) dx \quad 4$$

Or

Using partial fraction, evaluate

$$\int \frac{1}{x^4 - 1} dx \quad 4$$

(10)

- 30.** Solve the following homogeneous differential equation : 4

$$x dy - y dx = \sqrt{x^2 - y^2} dx$$

Or

Solve the following linear differential equation :

$$(1 - x^2) \frac{dy}{dx} - 2xy = \frac{1}{1 - x^2}$$

given $y = 0$, when $x = 1$. 4

- 31.** If a unit vector \vec{a} makes angle $\frac{\pi}{3}$ with \hat{i} , $\frac{\pi}{4}$ with \hat{j} and an acute angle with \hat{k} , then find \vec{a} and hence the components of \vec{a} . 4

Or

If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$. 4

- 32.** Solve the following linear programming problem graphically : 4

Maximize $Z = 4x + y$

subject to the constraints

$$x + y \leq 50$$

$$3x + y \leq 90$$

$$x \geq 0, y \geq 0$$

(11)

SECTION—D

- 33.** Verify $A(\text{adj } A) = (\text{adj } A)A = |A| I$ for the matrix

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 3 \end{pmatrix} \quad 6$$

Or

Solve the following system of linear equations using matrix method : 6

$$\begin{aligned} x + y + 2z &= 7 \\ 3x + 4y + 5z &= 5 \\ 2x + y + 3z &= 12 \end{aligned}$$

- 34.** Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere. 6

Or

Using definite integral, find the area of the region bounded by the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \quad 6$$

- 35.** Find the shortest distance between the lines whose vector equations are

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + 3\hat{j} + 2\hat{k})$$

and

$$\vec{r} = (4\hat{i} + 5\hat{j} + 6\hat{k}) + \mu(2\hat{i} + 3\hat{j} + \hat{k}) \quad 6$$

(12)

- 36.** A manufacturer has three machine operators A , B and C . The first operator A produces 1% defective items whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time and C is on the job for 20% of the time. A defective item is produced. What is the probability that it was produced by A ? 6

Or

Let X denote the number of hours you study during a randomly selected school day. The probability that X can take the values x has the following form, where k is a constant :

$$P(X = x) = \begin{cases} 0.1 & , \text{ if } x = 0 \\ kx & , \text{ if } x = 1 \text{ or } 2 \\ k(5 - x) & , \text{ if } x = 3 \text{ or } 4 \\ 0 & , \text{ otherwise} \end{cases}$$

Find the value of k . What is the probability that you study at least 2 hours, exactly 2 hours and at most 2 hours? 6
