

**2 0 2 3**

**STATISTICS**

*Full Marks : 100*

*Time : 3 hours*

*The figures in the margin indicate full marks for the questions*

*General Instructions :*

- (i) Write all the answers in the Answer Script.
- (ii) Attempt Part—A (Objective Questions) serially.
- (iii) Attempt all parts of a question together at one place.

( PART : A—OBJECTIVE )

( Marks : 50 )

SECTION—I

( Marks : 20 )

**1.** Choose and write the correct answer : 1×10=10

(a) If a random variable  $X$  takes the values 1, 2, 3 with probability  $P(X = r) = \frac{r}{6}$ ;  $r = 1, 2, 3$ . Then  $E(X)$  is equal

to

(i)  $\frac{7}{6}$

(ii)  $\frac{7}{3}$

(iii)  $\frac{3}{7}$

(iv)  $\frac{2}{7}$

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(b) If  $X$  is a random variable, then

(i)  $E(X^2) = \{E(X)\}^2$

(ii)  $E(X^2) = \{E(X)\}^2$

(iii)  $E(X^2) = \{E(X)\}^2$

(iv) None of the above

(c) If  $X$  follows binomial distribution with parameters  $n$  and  $p$ , then

(i)  $E \left( \frac{X}{n} \right) = p$

(ii)  $E \left( \frac{X}{n} \right) = p$

(iii)  $E \left( \frac{X}{n} \right) = p$

(iv) None of the above

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(d) If  $X$  follows Poisson distribution and  $P(X=0) = P(X=1) = a$ , then

(i)  $a = e^{-1}$

(ii)  $a = e^{-2}$

(iii)  $a = e^{-2}$

(iv)  $a = e$

(e) In Normal distribution

(i) Mean > Median

(ii) Mean > Mode

(iii) Mean < Median

(iv) Mean = Median = Mode

(f) Fisher's Ideal index number is

(i) the product of Laspeyres' and Paasche's index number

(ii) the GM of Laspeyres' and Paasche's index number

(iii) the AM of Laspeyres' and Paasche's index number

(iv) the addition of Laspeyres' and Paasche's index number

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(g) Time Reversal Test is satisfied by

- (i) Laspeyres' index number
- (ii) Paasche's index number
- (iii) Fisher's index number
- (iv) None of the above

(h) Time series consists of

- (i) one component
- (ii) two components
- (iii) three components
- (iv) four components

(i) If  $\sigma^2$  is the population variance, then which of the following is correct in case of SRSWR?

(i)  $V(\bar{x}) = \frac{\sigma^2}{n}$

(ii)  $V(\bar{x}) = \frac{\sigma^2}{n}$

(iii)  $V(\bar{x}) = \frac{\sigma^2}{n}$

(iv) None of the above

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(j) Changes that take place as a result of natural calamities are classified under the head

(i) secular movement

(ii) seasonal variations

(iii) cyclical variations

(iv) irregular variations

2. Fill in the blanks :

1×5=5

(a) If  $X$  is a random variable and  $a$  is any constant, then  $V(aX)$  \_\_\_\_\_.

(b) The recurrence relation for the probabilities of Poisson distribution is  $p(x)$  \_\_\_\_\_.

(c) Cost of living index number is used in adjustment of \_\_\_\_\_ allowances to compensate the rise in the price level.

(d) Natural cause is associated with \_\_\_\_\_ variation of time series.

(e) In case of SRSWOR,  $E(\bar{x})$  \_\_\_\_\_.

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3. State whether the following statements are *True* or *False* :

1×5=5

- (a) If  $X, Y, Z$  are random variables, then  
 $E(XYZ) = E(X)E(Y)E(Z)$ .
- (b) Normal distribution is unimodal.
- (c) Fisher's formula satisfies 'Factor Reversal Test'.
- (d) Fire in a factory is a seasonal component of time series.
- (e) Sample survey is superior than census.

SECTION—II

( Marks : 30 )

4. Answer the following questions :

3×10=30

- (a) If  $f(x) = \frac{1}{4}$ ,  $x = 2, 4, 8, 16$ , find  $E \frac{1}{X}$  where  
 $f(x) = P(X = x)$ .

- (b) Prove that

$$E[(X - c)^2] = \text{var}(X) + [E(X) - c]^2$$

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- (c) “The mean of a binomial distribution is 5 and standard deviation is 3.” State whether the statement is true or false. Give your answer with justification.
- (d) From a population of defective and non-defective items, 100 items are drawn at random. Find the probability of getting 2 defective items. Given that the population consists of 1% defective items.
- (e) Write any three uses of cost of living index number.
- (f) What is time reversal test? Show that Paasche’s index number formula does not satisfy this test.
- (g) Enumerate the objective of analysis of time series.
- (h) What is a time series? State its uses.
- (i) “Index numbers are economic barometers.” Explain.
- (j) From a large population, a random sample of 49 observations is taken. It is given that population SD is 3.5. Find the SE of the sample mean.

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( PART : B—DESCRIPTIVE )

( Marks : 50 )

Answer **four** questions, taking at least **one** from each Group

GROUP—A

5. (a) The random variable  $X$  is normally distributed with mean 2 and standard deviation 5. Find  $P(8 < X < 1)$ . Given that  $P\left(\frac{X-2}{5} < 0.2\right) = 0.5793$  and  $P\left(\frac{X-2}{5} < 2\right) = 0.9772$ .

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- (b) If  $X$  follows the binomial distribution

$$f(x) = {}^{18}C_x \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{18-x}; \quad x = 0, 1, 2, \dots, 18$$

Find the following :

(i)  $E\left(\frac{X-6}{10}\right)$

(ii)  $E(X-6)^2$

3+3=6

- (c) Write down the standard deviation of the following distribution : 1½

$$P(x) = \frac{e^{-4} 4^x}{x!}; \quad x = 0, 1, 2, 3, \dots$$



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6. (a) Prove that  $V(aX + b) = a^2V(X)$ , where  $a$  and  $b$  are constants. 4

- (b) If  $X$  and  $Y$  are two random variables, then prove that

$$V(X + Y) = V(X) + V(Y) + 2\text{cov}(X, Y) \quad 4\frac{1}{2}$$

- (c) If for a random variable  $X$ ,  $E(X) = 12$  and  $V(X) = 9$ , find the value of  $E(2X + 5)$  and  $V(2X + 5)$ . 4

GROUP—B

7. (a) Calculate the cost of living index number from the following data : 6½

	Price		
Items	Base year	Current year	Weight
Food	39	47	4
Fuel	8	12	1
Clothing	14	18	3
House Rent	12	15	2
Misc.	25	30	1

- (b) Show that Laspeyres' price index number and Paasche's index number do not satisfy Factor Reversal Test. 3+3=6

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8. (a) (i) What are the components of time series?
- (ii) State the various methods of measuring trend.  
Give two merits and one demerit of the least squares method.  $2+2+1\frac{1}{2}=7\frac{1}{2}$
- (b) State the components of a time series with which the following can be associated :  $1\times 5=5$
- (i) An era of prosperity
- (ii) Puja sales in a cloth store
- (iii) Recession
- (iv) Decrease in death rate due to advancement of medical services
- (v) Fire in a factory

GROUP—C

9. (a) Write any three demerits of Simple Random Sample (SRS).  $2\times 3=6$
- (b) A simple random sample of 10 households was drawn from a town of 1200 households. The number of persons in the selected houses were 2, 2, 3, 3, 4, 5, 6, 3, 3, 4. Estimate the number of persons in the town. 4
- (c) What do you mean by stratified random sampling?  $2\frac{1}{2}$

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10. (a) What are the differences between sampling error and non-sampling error? 6
- (b) In case of SRSWOR, prove that  $E(\bar{x}) = \bar{X}$ , i.e., the sample mean is an unbiased estimator of population mean.  $6\frac{1}{2}$

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